Mathematics: analysis and approaches - Standard level

PRACTICE PAPER 1 **Compiled by Sotiris Avdalas**

Topic 1: Number and algebra

Arithmetic sequences and series. Geometric sequences and series

1. [Maximum mark: 8] [without GDC]

Consider the arithmetic sequence 15, 19, 23, 27, …

(a) Find u_{24} and S_{24} [4]

(b) Find the sum of the first 40 terms. [2]

(c) Express the general term un in the form $u_n=an+b$. [2]

(d) Hence find the value of n given that $u_n=415$ [2]

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2. [Maximum mark: 7] [without GDC]

The 5th term of an arithmetic sequence is 74, while the 10th term is 64.

(a) Find the first term u_1 and the common difference d. [3]

(b) Find the general term u_n in terms of n. [1]

(c) Find the sum of the first *n* terms in terms of *n*. [1]

(d) Find the 40th term of the sequence and the sum of the first 40 terms. [2]

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3. [Maximum mark: 7] [without GDC]

Consider the arithmetic sequence 8, 15, 23, 30, …

(a) Find the number of terms which are less than 140. [3]

(b) Find the last term which is less than 140. [2]

(c) Find the sum of all terms which are less than 140. [2]

4. [Maximum mark: 8] [with GDC]

Consider the geometric sequence 3, 6, 12, 24, …

(a) Write down the first term u_1 and the common ratio r. [1]

(b) Find the 20th term of the sequence. [2]

(c) Find the sum of the first 10 terms. [2]

(d) Express the general term u_n in terms of n. [1]

(e) Hence find the value of n given that $u_n = 100\,633\,296$. [2]

5. [Maximum mark: 8] [with GDC]

Consider the geometric sequence 40, 10, 2.5, …

(a) Write down the first term u_1 and the common ratio r. [1]

(b) Find the 9th term of the sequence. [2]

(c) Find the sum of the first 9 terms. [2]

(d) Express the general term u_n in terms of n. [1]

(e) Hence find the value of n given that u_n= 0.000 038 146 97 [2]

6. [Maximum mark: 8] [with GDC]

Consider the geometric sequence 4, 12, 36, 108,⋯

(a) Find the number of terms which are less than 10 000. [3]

(b) Find the greatest term which is less than 10 000. [2]

(c) Find the greatest value of n such that Sn<10 000 [3]

7. [Maximum mark: 4] [without GDC]

The second term of an arithmetic sequence is 80. The sum of the first five terms of the arithmetic sequence is 390. Find the first term, u_1, and the common difference, d.

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8. [Maximum mark: 5] [without GDC]

The first and fourth terms of a geometric series are 20 and 2.5 respectively.

(a) Find the common ratio of the series [3]

(b) Find the sum to infinity of the series. [2]

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9. [Maximum mark: 9] [without GDC]

Calculate the sums

(a) 1+5+9 + .. + 197 (b) 2 + 8 + 32 + ... + 8192 (c) $\sum_{r=0}^{19}(4r-10)$

10. [Maximum mark: 8] [without GDC]

The sum of the first n terms of a sequence is given by $S_n = 2n^2 + n$, $n \in Z^+$.

(a) Find u_4. [3]

(b) Find an expression for u_n, the n^{th} term of the sequence. [3]

(c) Show that the sequence is arithmetic. [2]

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11. [Maximum mark: 8] [without GDC]

A table-shaped roof has 21 rows of tiles. The first row has 73 tiles and each subsequent row has two fewer tiles. How many tiles does the 15th row have and how many tiles does the roof have in total?

12. [Maximum mark: 8] [without GDC]

In an organism, there are initially 204800 bacteria. After 1 hour there are 102400 bacteria, after 2 hours there are 51200 bacteria , and in general the number of bacteria doubles every hour.

(a) How many bacteria will there be after 6 hours? [1]

But at the time when the bacteria were 3200, the organism suddenly deteriorated. The number of bacteria began to increase again so that every hour the number of bacteria tripled. This phenomenon lasted for 5 hours. We denote by the number of bacteria n hours after the moment of deterioration

(b) Show that the sequence is a geometric and find its first term and ratio. [2]

(c) Express the number of bacteria as a function of n. [2]

(d) How many bacteria will be present in the organism 3 hours after the time of deterioration? [2]

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